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THE INFLUENCE OF "SCALE-FREE" NETWORKS IN THE *1 SONATA*

Abstract: This article will discuss the influence of scale-free networks on my *1 sonata* (Sélection, 5th Dutilleux International Composition Compétition, 2003) for piano. An important feature of scale-free networks is that they are regulated by a small number of important nodes/hubs that are connected to many other sites. Following a power law distribution, research has found that a majority of nodes have only a few links, while a small minority of nodes have an enormous number of links, known as hubs. After a brief introduction, I will present four concepts and their applications in the *1 Sonata*.

Keywords: scale-free networks, complexity, second modernity, rich get richer, hubs, Six Degrees of Separation, Barabási.

This article will discuss the influence of scale-free networks on the composition of my *1 sonata* (Sélection, 5th Dutilleux International Composition Compétition, 2003) for piano. Popularized by Albert-László Barabási, scale-free networks are represented by simple laws that govern complex structures. In his

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words, "Just as diverse humans share skeletons that are almost indistinguishable, we have learned that these diverse maps follow a common blueprint. A string of recent breathtaking discoveries has forced us to acknowledge that amazingly simple and far-reaching natural laws govern the structure and evolution of all the complex networks that surround us" (Barabási 2002).

Research on scale-free networks suggest that networks are everywhere, from ecosystems to the internet (Cohen 2002; Dorogovtsev and Mendes 2003). An important feature of scale-free networks is that they are regulated by a small number of important hubs that are connected to many other sites (Barabási and Albert 1999). Following a power law distribution, research has found that a majority of nodes have only a few links, while a small minority of nodes have an enormous number of links, known as hubs. This is dramatically different than the previous 50 years of research on complex networks, which were understood to be random with each node having approximately the same number of links (see Figure 1). What recent research has found is that instead of following a bell-shaped distribution (like the height of the majority of the world's population), many networks feature a power law distribution (or finding the equivalent of lots of folks who are 100 feet tall) (Barabási and Bonabeau 2003).

These networks are said to be *scale-free* due to the extreme difference in links between hubs and nodes, which are essentially encompassed by logarithmic, rather than linear, functions. In a random network, most nodes have about the same number of links, while scale-free networks lack a characteristic scale as they are too unpredictable. Perhaps a more precise label would be a *rich get richer* network, just as a store with better location and access with larger quantity of low-priced items will attract more customers (links are more likely to attach to nodes with more connections).

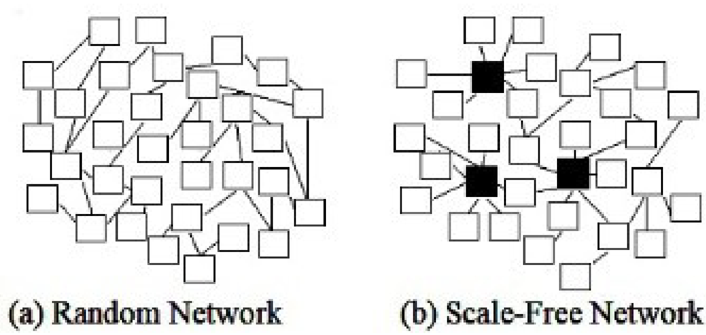

Figure 1. Random networks (L) feature nodes having about the same number of links, while scale-free networks (R) are regulated by a small number of important hubs, each with many links.

Concepts and Applications

Next I will present four concepts and how they were applied in the *1 Sonata*, beginning with the idea of robust hubs which were interpreted in various ways.

CONCEPT ONE: Hubs. The highway system in the US is an example of a random network distribution that consists of nodes which have approximately the same number of links, and as such each road is statistically equal to each other. In comparison, consider the airport system in which the majority of hubs have just a few connections, while others will have a tremendous number of links (Caldarelli 2007).

APPLICATION 1: Six hubs were distributed over the second and third movements, which were interpreted according to the size and activity at the following airports: Seattle/Tacoma, Las Vegas/MacCarran, Dallas/Fort Worth, Atlanta Hartsfield, Chicago O'Hare, New York/JFK. Musically, these hubs function similar to real-world airports – as a place of respite (from flight or musical activity) and a period of maintenance (refueling, replacing parts or reflection on the preceding musical arguments) and thus carry a static quality. Although each airport is considered a large hub (in 2017 each ranked within the top 10 of US airports), in the *1 sonata* the complexity of events occurring at arrival and departure varied as a function of time (less flights at 1am versus 1pm). Further, the idea of an airport was invoked by diverting all air traffic onto their respective runways, or rather into a limited area represented by the rhythmically free repetition of octaves in all registers. APPLICATION 2. The idea of hubs was used to generate complex and lengthy musical passages through simple generative processes. In this application, the idea of a hub was used to identify a space (measure), into which was placed rhythmic figures. These figures were developed through cyclical procedures, in which integers were used to generate meter and iterative values, even including embedded tuplets. This information resulted in *base* rhythmic sequences. These sequences were then either used as *cantus firmi* which could be applied verbatim into the composition, or could be used as structural foundations upon which further integral iterations could build uniform or non-uniform second-level gestures. Thus given a base rhythm (hub), the resultant overlain iterations represent either a) *important hubs with many links* when the overlain iterations are many, or b) *statistically-equal nodes with only a few links* when the overlain iteration equals the basic rhythm (see Figure 2). APPLICATION 3. The idea of an important hub was combined with the notion of a pitch matrix. In this application, sequences of rhythmic *cantus firmi* were built and segmented into small cells, analogous to pitch sequences.

Then the rhythmic matrix was built by transposing each cell, so that each transposition features a progressive lengthening of rhythmic value.

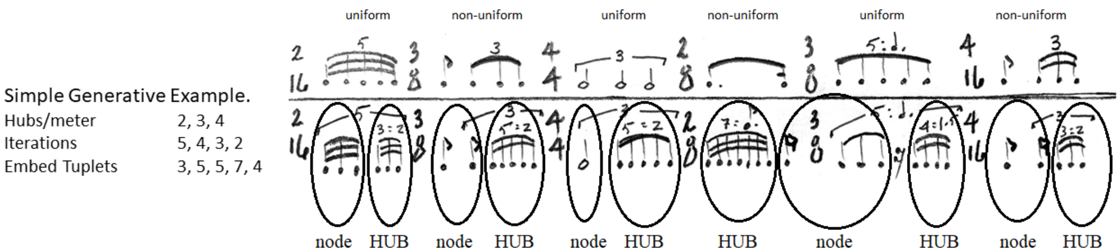

Figure 2. The idea of hubs were used to generate complex and lengthy musical passages through simple generative processes. In Figure 2, the cantus firmus (upper phrase) was built through cyclical generative processes which were either uniform or non-uniform. Then below, further iterations were used to develop higher level tuplets. In this figure, the idea of hubs had two interpretations: 1) in the upper passage, each measure is considered to be a hub, while 2) in the lower passage, the higher level embedded tuplets are considered to be hubs (a node with many links), while the non-altered notes are considered to be nodes.

CONCEPT TWO: Birth of a Scale-Free Network or *rich get richer*. The growth of a scale-free network tends to establish links on existing nodes that already have many other connections. Both growth and preferential attachment eventually lead to a system being dominated by hubs with an enormous number of links. The result in the real world is that already prominent nodes become more robust (links, money, prestige, etc.), thus assuring that the system is dominated by the rich and powerful (ex. Walmart) (Barabási 2009; Gladwell 2008). APPLICATION 4. In the *1 sonata*, most hubs began as relatively complex, which exponentially increased their density through the generative process (growth of the network). Meanwhile, less significant nodes featured less growth and in some cases, decay.

CONCEPT THREE: Six Degrees of Separation. Networks sharing a "small-world" property – for instance, it has been found that a path of just three reactions will connect almost any pair of chemicals in a cell. Further, society is fragmented into clusters of individuals having similar characteristics (such as income or interests) and clustering is a general feature of many other types of networks, including the US Power Grid to biological neural networks. The link between scale-free topology and clustering occurs when small, tightly interlinked clusters of nodes are connected into larger, less cohesive groups (Williams et al. 2002). APPLICATION 5. The identification of a gesture and its repetition was mapped over time. Initially nine species of gesture were planned, but this resulted in overpopulation within the allocated space. As predicted, the available resources were used up too quickly, which led to starvation. This led to a reduction of species to seven, resulting in a distribution in which the resources were stable, providing enough room for growth.

CONCEPT FOUR: Attack on Hubs. Both the internet and biological systems are robust enough to survive an attack without major disruption. For random networks, if a critical fraction of nodes are removed, these systems break into tiny, non-communicating islands. However, scale-free networks are different. Up to 80% of randomly selected internet routers can fail and the remaining ones will still form a compact cluster in which there will still be a path between any two nodes. The robustness is due to an INHOMOGENEOUS topology. In this type of system, the removal of small hubs is ok, but the removal of large hubs may crash a system. It seems that a 5%to 15% simultaneous elimination of all hubs can crash a system, but that by protecting the largest hubs, the system may still function against such a coordinated attack (Cohen, Havlin, and ben-Avraham 2006). APPLICATION 6. The 1st and 2nd movements use an inhomogeneous topology of a quasi-*cantus firmus* to protect against a system attack. However, the 3rd movement loosely simulates a system attack during the generative process, with the result being that the number of nodes are greatly diminished.

Musically, nodes can be interpreted in many ways. One useful application involved defining how phrases begin and end. Since functional tonal harmony and expected phrase lengths, such as found in common-practice music, are not used in the *1 sonata*, these small nodes are important structural markers which are defined by: (a) silence, (b) complex sonorities, (c) placement in extreme registers, (d) octave sonorities, (e) sustained dissonant sonorities and/or (f) the good continuation leading to culmination of the vertical/horizontal motion. In Figure 3, the nodes used to begin and end phrases are identified as a-c on the musical excerpt (see Figure 3).

At the beginning of the musical excerpt, there is a nodal cadence that features properties € & (f); a sustained dissonant harmony, along with the good culmination of rhythmic motion. Then a single phrase in two parts appear. At the elision between the 1st and 2nd parts we have the articulation of events that occur in (c) extreme registral separation between the right and left hands. Then a somewhat proper cadence occurs at the end of the musical excerpt, in which (b) the articulation of a complex sonority is followed by (a) significant silence, which helps to define local closure.

CONCEPT FIVE: Integration of Scale-Free Topology with Modular Structure. This last concept rests upon a hierarchical network integrating scale-free topology with modular structure. An example was proposed that consisted of a simple heuristic model of metabolic organization (Oltvai and Barabási 2002). In such a network, the starting point is a small cluster of four densely linked nodes. Then, three replicas were generated. The three external nodes of each replication were linked to the central node of the first cluster, and thus obtaining a large 16 node cluster. Similarly, we can generate three replicas of this 16-node module and connect the peripheral nodes to the central node of the first cluster.

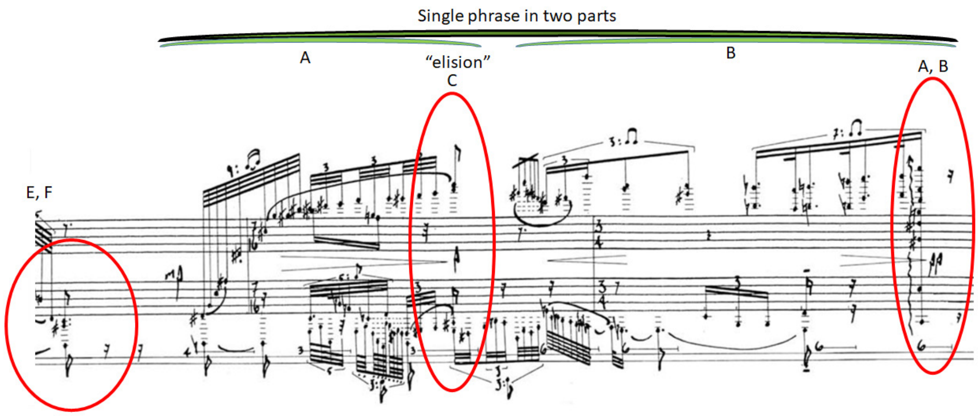

Figure 3. Inhomogeneous topology of a quasi-cantus firmus to protect against a system attack. In Figure 3, three nodes are identified: the first ends the previous phrase, while the second is a weak cadence that closes the first phrase (antecedent?) of a two-phrase group (quasi-period structure).

This process of replication at larger scales can be repeated indefinitely, with each repetition quadrupling the number of nodes in the system (Ravasz et al. 2002). APPLICATION 7. Combined with the concept of cosmological universal expansion, the idea was to expand from a highly-active and densely-clustered cellular topology to one in which the growth of expansion would exceed Friedmann's assumptions and continue to expand at a rate greatly exceeding current estimations of no-static universal growth (Nussbaumer 2014). This expansion utilized a power law degree distribution = 2.26, which was applied to temporal units. Then regarding pitch; in an expanding universe, scientists have observed what is known as a red shift, which implies that to any observer in the universe, the frequencies of the galaxies are seen to be decreasing (because to all observers, anywhere in the universe, all galaxies are moving away from all observers) (Nussbaumer and Bieri 2009). Therefore, after an initial rapid expansion (pitch separation), the phrase onset identifies widely spaced strata which gradually narrow over time.

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